

MATHEMATICS 125 TEST CHAPTER 12 --- FALL 2004 (EXPONENTIAL AND LOG FCN)
INSTRUCTOR: ANNE SISWANTO; TOTAL POINTS: 100; TIME: 70 MINUTES

DIRECTION: GRAPHING CALCULATORS ARE NOT ALLOWED. SHOW ALL WORKS ON THE TEST PAPER FOR FULL CREDIT

QUESTION 1 (10 POINTS)

Perform the following operations.

- a. <2> Write in exponential notation;

$$\log_6 36 = 2$$

$$\underline{\underline{6^2 = 36}}$$

- b. <2> Write using logarithmic:

$$2^{-5} = \frac{1}{32}$$

$$\underline{\underline{\log_2 \left(\frac{1}{32} \right) = -5}}$$

c. <2> Condense: $3\log_b y + \frac{1}{2}\log_b x - 2\log_b z = \underline{\underline{\log_b \frac{y^3 x^{\frac{1}{2}}}{z^2}}}$

d. <2> Calculate: $\log_5 16 = \frac{\log 16}{\log 5} = \underline{\underline{1.723}}$

e. <2> Expand: $\log_2 \frac{x^3 \sqrt{x}}{yz} = \log_2 \frac{x^{3\frac{1}{2}}}{yz} = \underline{\underline{3\frac{1}{2} \log_2 x - \log y - \log z}}$

QUESTION 2 (15 POINTS)

Solve for x , you may use a calculator:

a. <4> $\log_2 x = 3$

$$x = 2^3$$

$$\underline{\underline{x = 8}}$$

b. <4> $\log_8 4 = x$

$$x = \frac{\log 4}{\log 8}$$

$$x = \frac{2}{3} = \underline{\underline{0.667}}$$

c. <4> $\log_{10} 0.01 = x$

$$x = \log_{10} 10^{-2}$$

$$\underline{\underline{x = -2}}$$

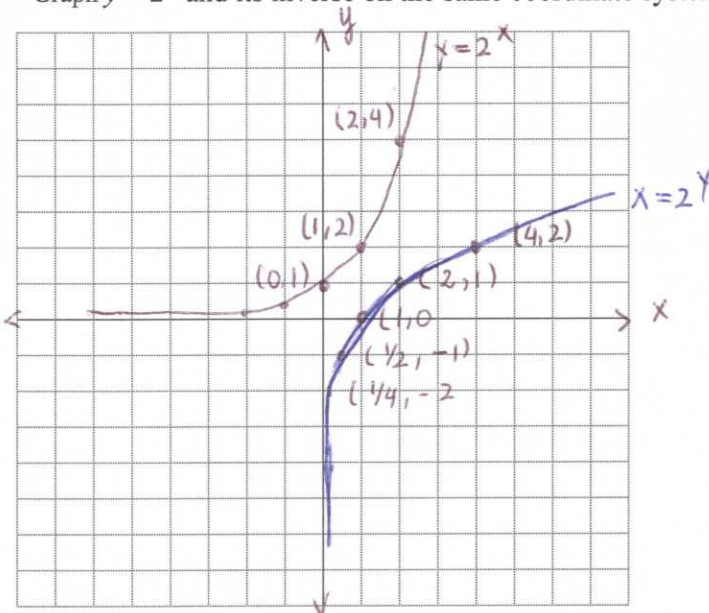
d. <3> $\log_{10} x = 2.8993$

$$x = 10^{2.8993}$$

$$\underline{\underline{x = 793.05}}$$

QUESTION 3 (8 POINTS)

Graph $y = 2^x$ and its inverse on the same coordinate system. Then give an equation of the inverse.



x	$y = 2^x$	Points
-2	$2^{-2} = 1/4$	$(-2, 1/4)$
-1	$2^{-1} = 1/2$	$(-1, 1/2)$
0	1	$(0, 1)$
1	2	$(1, 2)$
2	4	$(2, 4)$

To graph Inverse, reverse the x and y coordinates
 $(1/4, -2)$
 $(1/2, -1)$
 $(1, 0)$
 $(2, 1)$
 $(4, 2)$

Equation of inverse:
 $\underline{\underline{x = 2^y}}$ or $\underline{\underline{y = \log_2 x}}$

QUESTION 4 (10 POINTS)

Find $f^{-1}(x)$ for the following function if it exists.

a. <5> $f(x) = 6x + 5$

$$y = 6x + 5$$

$$x = 6y + 5$$

$$x - 5 = 6y$$

$$\frac{x-5}{6} = y$$

$$\underline{\underline{f^{-1}(x) = \frac{x-5}{6}}}$$

$$b. <5> h(x) = \frac{5}{x-3}$$

$$y = \frac{5}{x-3}$$

$$x = \frac{5}{y-3}$$

$$xy - 3x = 5$$

$$\frac{xy}{x} = \frac{5+3x}{x}$$

$$y = \frac{5+3x}{x}$$

$$\underline{\underline{f^{-1}(x) = \frac{5+3x}{x}}}$$

QUESTION 5 (37 POINTS)

Solve for x:

$$a. <7> 2 \log x = -8$$

$$\log x = -4$$

$$x = 10^{-4}$$

$$\underline{\underline{x = 0.0001}} \quad \text{a soln}$$

$$b. <7> 5^x = 3$$

$$x \frac{\log 5}{\log 5} = \frac{\log 3}{\log 5}$$

$$x = \frac{\log 3}{\log 5} = \underline{\underline{0.683}}$$

$$c. <7> 4^{3x+2} = 5$$

$$(3x+2) \log 4 = \log 5$$

$$3x+2 = \frac{\log 5}{\log 4}$$

$$3x+2-2 = 1.161-2$$

$$\frac{3x}{3} = \frac{-0.839}{3}$$

$$\underline{\underline{x = -0.280}}$$

d. <8> $\log_2(x-2) + \log_2 x = 3$

$$\begin{aligned} \log_2 x(x-2) &= 3 \\ 2 & \\ x(x-2) &= 8 \\ x^2 - 2x &= 8 \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \end{aligned}$$

$$\begin{aligned} x &= 4 \quad \text{or} \quad x = -2 \\ \underline{\underline{(soln)}} & \quad \underline{\underline{(not a soln)}} \end{aligned}$$

e. <8> $\log_4(x+6) - \log_4 x = 2$

$$\begin{aligned} \log_4 \frac{x+6}{x} &= 2 \\ 4 & \\ \frac{x+6}{x} &= 16 \\ x+6 &= 16x \\ 6 &= 15x \end{aligned}$$

$$\begin{aligned} x &= \frac{6}{15} \\ x &= \underline{\underline{\frac{2}{5}}} \quad (\text{a solution}) \end{aligned}$$

QUESTION 6 (12 POINTS)

Suppose you deposited \$1200 in an account with an annual interest rate of 8% compounded quarterly.

a. <4> Find an equation that gives the amount of money in the account after t years.

$$\begin{aligned} A &= P \left(1 + \frac{r}{m}\right)^{mt} \\ A &= 1200 \left(1 + \frac{0.08}{4}\right)^{4t} = 1200(1 + 0.02)^{4t} = \underline{\underline{1200(1.02)^{4t}}} \quad \therefore A(t) = 1200(1.02)^{4t} \end{aligned}$$

b. <4> Find the amount of money in the account after 8 years.

$$A(8) = P(1.02)^{4 \cdot 8} = (1.02)^{32}$$

$$A(8) = \underline{\underline{2261.45}}$$

\therefore The amount of money after 8 years = \$2261.45

c. <4> How many years will it take for the account to contain \$2400?

$$\begin{aligned} 2400 &= 1200(1.02)^{4t} \\ 2 &= 1.02^{4t} \\ \frac{\log 2}{4 \log 1.02} &= \frac{4t \log 1.02}{4 \log 1.02} \end{aligned}$$

$$t = \underline{\underline{8.75}}$$

\therefore It will take 8.75 years to have \$2,400.00

QUESTION 7 (8 POINTS)

The band U2 recently performed and sound measurements of 105 dB were recorded. What is the intensity of such sounds?

$$L = 10 \log \frac{I}{I_0}$$

$$105 = 10 \log \left(\frac{I}{10^{-12}}\right)$$

$$\begin{aligned} 10.5 &= \log \frac{I}{10^{-12}} \\ 10 & \quad 10 \end{aligned}$$

$$10^{10.5} = \frac{I}{10^{-12}}$$

$$I = 10^{-1.5}$$

$$\underline{\underline{I = 0.0316}}$$

\therefore The intensity of such sound is 0.0316 W/m²